element of choice in the circles γ and γ' does not affect the value of the integral. We thus obtain the result—

The integral

$$\frac{1}{2\pi}\iint\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}\,dxdy,\right.$$

where the integration is taken over the interior of any periodic orbit in the restricted problem of three bodies, has the value

$$-\frac{\Gamma}{\pi}, \text{ or } -1-\frac{\Gamma}{\pi}, \text{ or } -2-\frac{\Gamma}{\pi},$$

according as the periodic orbit encloses both, one, or none of the bodies S *and* J*; where* Γ *is the angle through which the whole system composed of* S, J, *and* P *turns in space during one period of the periodic orbit.*

1902 *January* 24.

On the Images formed by a Parabolic Mirror. First Paper: The Geometrical Theory. By H. C. Plummer, M.A.

1. *Introduction.*—As is well known the staple work of the University Observatory at Oxford during the past few years has been the execution of its share of the Astrographic Chart. At the same time lines of investigation suggested by this work have not been neglected, and much has been done in the discussion of the systematic errors to which photographs are liable. As an example of such researches reference may be made to Professor Turner's papers on the optical distortion of the photographic doublet (*Monthly Notices*, vol. lix. p. 438).

Soon after I became associated as assistant with the observatory Professor Turner suggested that, as I had already paid some attention to the forms of the images produced by a reflecting telescope (*A. J.*, No. 435) I should extend and as far as possible complete this study. Accordingly I undertook the direct comparison of a plate kindly lent by Dr. Isaac Roberts with one of our own astrographic plates. This task is proceeding and will, I hope, shortly be finished. In the meantime a comparison of my own earlier paper with various other theoretical papers revealed apparent discrepancies which it is desirable to remove by a thorough revision of this part of the subject. The results of this investigation are independent of the actual comparison of the two plates, and it seems convenient therefore to publish them separately.

2. My earlier paper, "On the Star Image formed by a Parabolic Mirror," was published in the *Astronomical Journal*,

No. 435, simultaneously with one by Mr. J. M. Schaeberle. Previously, in 1887, the subject had been discussed by General Tennant (*Monthly Notices*, vol. xlvii. p. 244). A fourth paper, by Mr. C. W. Crockett (*Astrophysical Journal*, vol. vii. p. 362), will also receive notice. Curiously enough the first two and the last of these papers all bear the date 1898 March. Differences of notation and variety of method have made it necessary to collate these papers with considerable care. The outcome of the examination may be briefly indicated here. In General Tennant's paper an unfortunate slip is pointed out which seems to have escaped detection hitherto, and which makes his final results only an approximation to the truth. I have found that Mr. Schaeberle's formulæ are right, though left in a rather inconvenient form, if we correct an apparent misprint. On the other hand his figures of the image formed by different zones of the mirror are certainly incorrect in detail. In Mr. Crockett's results, so far as they go, there is nothing calling for criticism; on the contrary, I have found his curves, evidently drawn with great care, most interesting. In my own paper rigorous expressions were found for the equations of the reflected ray. The further development of the theory was based on a clearly defined approximation which led in a remarkably simple manner to results sufficiently in accord with the actual appearance of star images. The degree of approximation adopted is easily examined, but no examination was made before and the omission will be rectified in the present paper. The analysis will also be extended so as to consider the form of the image in a plane close to, but not coincident with, the focal plane, a step which has not hitherto been taken in any paper, so far as I am aware.

3. *General Theory.*—I retain as far as possible the notation of my previous paper. Let the equation of the surface of the mirror be

$$4fx = y^2 + z^2 = r^2.$$

Let the plane ZOX be the principal plane, *i.e.* the plane containing the axis of the mirror and the ray incident at O, the vertex, and let θ be the angular distance of the star from the centre of the field, being in the positive direction from OX. The direction cosines of the normal at P, the point of incidence (x, y, z), are

$$\frac{2f}{\sqrt{4f^2+r^2}}, \quad \frac{-y}{\sqrt{4f^2+r^2}}, \quad \frac{-z}{\sqrt{4f^2+r^2}}.$$

We may then put

$$r = 2f \tan \tfrac{1}{2}v, \; y = r \sin \phi, \; z = r \cos \phi.$$

If F is the focus, v is the angle OFP and ϕ is the angle

between the planes POX and ZOX. The direction cosines of the normal become

$$+\cos\tfrac{1}{2}v, \quad -\sin\tfrac{1}{2}v\sin\phi, \quad -\sin\tfrac{1}{2}v\cos\phi.$$

If χ is the angle between the incident ray and the normal we have

$$\cos\chi = \cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta$$

Let (λ, μ, ν) be the direction cosines of the reflected ray. Now the direction cosines of the incident and reflected rays may also be regarded as the coordinates of points at unit distance from O along lines parallel to those rays. And the direction cosines of the normal multiplied by $\cos\chi$ are the coordinates of a point at a distance $\cos\chi$ from O along a line parallel to the normal. But this point is clearly midway between the other two, and by expressing this fact we obtain

$$\begin{aligned}
\lambda + \cos\theta &= 2\cos\tfrac{1}{2}v\left[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta\right] \\
\mu &= -2\sin\tfrac{1}{2}v\sin\phi\left[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta\right] \\
\nu + \sin\theta &= -2\sin\tfrac{1}{2}v\cos\phi\left[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta\right].
\end{aligned}$$

Hence

$$\left.\begin{aligned}
\lambda &= \cos v\cos\theta - \sin v\sin\theta\cos\phi \\
\mu &= -\sin v\cos\theta\sin\phi + \sin^2\tfrac{1}{2}v\sin\theta\sin 2\phi \\
\nu &= -\sin v\cos\theta\cos\phi + \sin^2\tfrac{1}{2}v\sin\theta\cos 2\phi - \cos^2\tfrac{1}{2}v\sin\theta
\end{aligned}\right\} \quad (1)$$

4. Now let the reflected ray, whose direction cosines have just been found, meet the focal plane in the point (ξ, η, ζ). Then,

$$\xi = f, \qquad \eta = y + \frac{\mu}{\lambda}(f - x), \qquad \zeta = z + \frac{\nu}{\lambda}(f - x).$$

Remembering that

$$f - x = f(1 - \tan^2\tfrac{1}{2}v) = f\cos v\sec^2\tfrac{1}{2}v\,;$$

we then find

$$\begin{aligned}
\lambda\eta &= 2f\tan\tfrac{1}{2}v\sin\phi\,(\cos v\cos\theta - \sin v\sin\theta\cos\phi) \\
&\qquad + f\cos v\sec^2\tfrac{1}{2}v\,(-\sin v\cos\theta\sin\phi + \sin^2\tfrac{1}{2}v\sin\theta\sin 2\phi) \\
&= -f\tan^2\tfrac{1}{2}v\sin\theta\sin 2\phi. \\
\lambda\zeta &= 2f\tan\tfrac{1}{2}v\cos\phi\,(\cos v\cos\theta - \sin v\sin\theta\cos\phi) \\
&\qquad + f\cos v\sec^2\tfrac{1}{2}v\,(-\sin v\cos\theta\cos\phi + \sin^2\tfrac{1}{2}v\sin\theta\cos 2\phi \\
&\qquad - \cos^2\tfrac{1}{2}v\sin\theta) = -f\tan^2\tfrac{1}{2}v\sin\theta\cos 2\phi - f\sin\theta.
\end{aligned}$$

The co-ordinates may therefore be written in the form

$$\left.\begin{aligned}
\eta &= -\frac{f\tan\theta}{\cos v}\cdot\frac{\tan^2\tfrac{1}{2}v\sin 2\phi}{1 - \tan v\tan\theta\cos\phi} \\
\zeta &= -\frac{f\tan\theta}{\cos v}\cdot\frac{1 + \tan^2\tfrac{1}{2}v\cos 2\phi}{1 - \tan v\tan\theta\cos\phi}
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (2)$$

These simple expressions make a very convenient basis for a study of the aberration in the focal plane of a parabolic mirror.

5. Let us now put

$$d = -f \tan\theta \sec v, \qquad a = \tan^2 \tfrac{1}{2}v, \qquad b = \tan v \tan\theta.$$

Then the equations found for the co-ordinates may be written

$$\left.\begin{aligned} \frac{\eta}{d} &= \frac{a \sin 2\phi}{1 - b \cos\phi} \\ \frac{\zeta}{d} &= \frac{1 + a \cos 2\phi}{1 - b \cos\phi} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

and these may be regarded as equations in terms of a third variable ϕ of the geometrical image formed in the focal plane by a circular zone of the mirror. They are in the form most convenient for the discussion of the properties of the curve which they represent, and make it easy to plot the curve for any assigned values of the constants. It is merely of mathematical interest to eliminate ϕ and so obtain the ordinary Cartesian equation. It is convenient first to move the origin from the centre of the field along the axis of ζ to the point $\zeta = d(1-a)$. When this is done the result of elimination is :

$$\begin{aligned} &(\eta^2+\zeta^2)[\eta^2\{(1-a)b^2+2a\}^2+4a^2(1-b^2)\zeta^2] \\ &\quad + 2d\zeta[\eta^2\{b^2(1-a)^2(b^2\overline{1-a}+2a)+2aJ\} - 4a^2\zeta^2(b^2\overline{1-a}+2a)] \\ &\quad + d^2J\{b^2(1-a)^2\eta^2-4a^2\zeta^2\} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4) \end{aligned}$$

where

$$J = b^2(1-a)^2 - 4a^2.$$

The curve is therefore a quartic.

6. We can now draw from (2) some simple inferences with regard to the properties of the curves formed by the rays reflected at a circular ring of the mirror. In the first place we see, by changing the sign of ϕ, that the curve is symmetrical about a radius through the centre of the field, a fact which is obvious *a priori* and from the form of equation (4). On the trace of the principal plane or axis of symmetry we have the following scheme of values :—

$$\phi = 0, \quad \eta = 0, \quad \zeta = -f\frac{\tan\theta \,.\, \sec v \sec^2\frac{1}{2}v}{1 - \tan v \tan\theta} = d \,.\, \frac{1+a}{1-b} \ldots \quad (A)$$

$$\phi = \frac{\pi}{2}, \quad \eta = 0, \quad \zeta = -f \tan\theta \,.\, \sec^2\tfrac{1}{2}v = d(1-a) \quad \ldots \quad (B)$$

$$\phi = \pi, \quad \eta = 0, \quad \zeta = -f\frac{\tan\theta \,.\, \sec v \sec^2\frac{1}{2}v}{1 + \tan v \tan\theta} = d \,.\, \frac{1+a}{1+b} \ldots \quad (C)$$

$$\phi = \frac{3\pi}{2}, \quad \eta = 0, \quad \zeta = -f \tan\theta \,.\, \sec^2\tfrac{1}{2}v = d\,(1-a) \quad \ldots \quad (B)$$

It may be remarked that investigations confined to the principal plane are quite inadequate, inasmuch as they fail to explain the genesis of the image in the principal plane itself. The point B is clearly a double point lying on the two branches of the curve which correspond to $\phi=\frac{\pi}{2}$ and $\phi=\frac{3\pi}{2}$. This also is shown by the Cartesian equation, which gives for the tangents at the point

$$\left.\begin{array}{l} b(1-a)\eta\pm 2a\zeta=0 \\ \text{or} \quad \tan\theta\,.\,\eta\pm\tan\tfrac{1}{2}v\,.\,\zeta=0 \end{array}\right\} \quad \ldots \quad \ldots \quad (5)$$

This would indicate that the tangents are equally inclined to the axes when $v=2\theta$. It will be noticed, however, that J, the numerical coefficient of the terms of the lowest degree, vanishes at the same time. It is necessary to observe therefore that the terms of the third degree possess the terms of lowest degree as a factor when the coefficient vanishes, and continuity is thus preserved. I have purposely thrown the terms of the third degree into a form which reveals this fact on inspection. Thus when $v=2\theta$ there is a triple point through which pass the branches corresponding to $\phi=\frac{\pi}{2}$, $\phi=\pi$, and $\phi=\frac{3\pi}{2}$. When $v>2\theta$, the points BCA are in order of increasing distance from the centre of the field and from the point I corresponding to the ray reflected at the vertex of the mirror.

7. It is important now to examine the maximum and minimum values of the coordinates. For a critical value of η

$$2\cos 2\phi\,(1-\tan v\tan\theta\cos\phi)=\tan v\tan\theta\sin\phi\sin 2\phi$$

$$\therefore\ 2\cos 2\phi-\tan v\tan\theta\cos 2\phi\cos\phi=\tan v\tan\theta\cos\phi$$

$$\therefore\ \cos 2\phi=\tan v\tan\theta\cos^3\phi \quad \ldots \quad \ldots \quad (6)$$

Now $\tan v\tan\theta = b$ is always small in a practical case. To take an extreme instance, one of Dr. Common's mirrors has a focal length of 3·7 feet and an aperture of 1·7 feet. For the outside ring $v = 13°$, and if $\theta = 2\tfrac{1}{2}°$ b only amounts to ·01. Hence we are justified in taking $\phi = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$ as the solution of (6), though a closer approximation to the roots can easily be made if desired. The points where η is a maximum or minimum are therefore given by

$$\phi=\frac{\pi}{4} \text{ or } \frac{7\pi}{4},\ \eta=\pm d\,.\,\frac{a}{1-b/\sqrt{2}},\ \zeta=d\,.\,\frac{1}{1-b/\sqrt{2}} \quad \ldots \quad (\text{E, E}')$$

$$\phi=\frac{3\pi}{4} \text{ or } \frac{5\pi}{4},\ \eta=\pm d\,.\,\frac{a}{1+b/\sqrt{2}},\ \zeta=d\,.\,\frac{1}{1+b/\sqrt{2}} \quad \ldots \quad (\text{D, D}')$$

Since b is small the width of the curve at the points where ζ has these two values is practically the same, viz. $2ad$.

Again, for a critical value of ζ,

$$-2\tan^2\tfrac{1}{2}v\sin 2\phi\,(1-\tan v\tan\theta\cos\phi)$$
$$=\tan v\tan\theta\sin\phi\;(1+\tan^2\tfrac{1}{2}v\cos 2\phi)\quad\ldots\quad\ldots\quad\ldots\quad(7)$$

This is satisfied by $\sin\phi=0$, *i.e.* $\phi=0$ or π. The curve at the points A and C crosses the axis of ζ at right angles. The factor $\sin\phi$ being omitted, the equation reduces to

$$2\tan^2\tfrac{1}{2}v\tan v\tan\theta\cos^2\phi-4\tan^2\tfrac{1}{2}v\cos\phi$$
$$-\tan v\tan\theta\,(1-\tan^2\tfrac{1}{2}v)=0$$

or
$$2ab\cos^2\phi-4a\cos\phi-b\,(1-a)=0$$
$$\therefore\; 2ab\cos\phi=2a\pm\surd[4a^2+2a(1-a)b^2]$$
$$\therefore\; 2ab\cos\phi=2a\pm[2a+\tfrac{1}{2}b^2(1-a)-\cdot\cdot\,]$$

The upper sign is impossible, for it would make $\cos\phi>1$. The lower sign, if higher powers of b be neglected, gives

$$\cos\phi=-b\,(1-a)/4a=-\tan\theta/2\tan\tfrac{1}{2}v.$$
$$\therefore\phi=\pi\pm\cos^{-1}\frac{\tan\theta}{2\tan\frac{1}{2}v}\quad.\quad.\quad.\quad(\mathrm{H},\,\mathrm{H}')$$

8. These values of ϕ give minimum values of ζ which become impossible when v is small, the limit occurring when $v=\theta$ approximately. After the disappearance of these minima as the centre of the mirror is approached the true minimum is given by $\phi=\pi$. It is then theoretically important to determine v so as to make ζ an absolute minimum. Now

$$\zeta=-f\tan\theta\frac{\sec^2\frac{1}{2}v}{\cos v+\sin v\tan\theta}.$$

Hence for a minimum

$$\sec^2\tfrac{1}{2}v\tan\tfrac{1}{2}v(\cos v+\sin v\tan\theta)=\sec^2\tfrac{1}{2}v(-\sin v+\cos v\tan\theta)$$
$$\therefore\;\tan\tfrac{1}{2}v(1-\tan^2\tfrac{1}{2}v+2\tan\tfrac{1}{2}v\tan\theta)+2\tan\tfrac{1}{2}v$$
$$-(1-\tan^2\tfrac{1}{2}v)\tan\theta=0$$
$$\therefore\;\tan\theta=\frac{3\tan\frac{1}{2}v-\tan^3\frac{1}{2}v}{1-3\tan^2\frac{1}{2}v}=\tan\tfrac{3}{2}v$$
$$\therefore\; v=\tfrac{2}{3}\theta.$$

This value of v leads to

$$\zeta=-f\sin\theta\sec^3\tfrac{1}{3}\theta.$$

Also it is easy to see that the maximum value of ζ corresponds to $\phi=0$ and is by (A)

$$\zeta=-f\sin\theta\,.\,\sec^2\tfrac{1}{2}v\sec(v+\theta),$$

which again is greatest when v corresponds to the edge of the mirror. The total length of the complete geometrical image is therefore

$$l=f\sin\theta\,[\sec^2\tfrac{1}{2}v\sec(v+\theta)-\sec^3\tfrac{1}{3}\theta]\quad.\quad.\quad.\quad(8)$$

and if powers and products of v and θ of the fifth order be neglected the expression obtained becomes

$$l=\tfrac{1}{12}f\theta(3v+2\theta)^2.$$

But this expression is of purely theoretical interest and does not apply strictly to a reflector as adapted for photographic use, since the central part of the mirror is cut off by the plate-holder and thus a part of the image is lost (*see* §15).

9. *On General Tennant's Paper* (*Monthly Notices*, vol. xlvii. p. 244).—I follow here General Tennant's own notation. He writes down (p. 250) the equations of the reflected ray thus: in the plane XY_o

$$y_o-av=\frac{4c-4v-cv^2}{4+4cv-v^2}\left(x-\frac{av^2}{4}\right)$$

and in the plane XZ_o

$$Z_o=\frac{(av^2}{4}-x)\tan\alpha\sin\phi.$$

I say that the latter equation is incorrect. Simple geometrical considerations would show that this is so, but perhaps the most satisfactory method will be to find the right equation which ought to be substituted for the preceding.

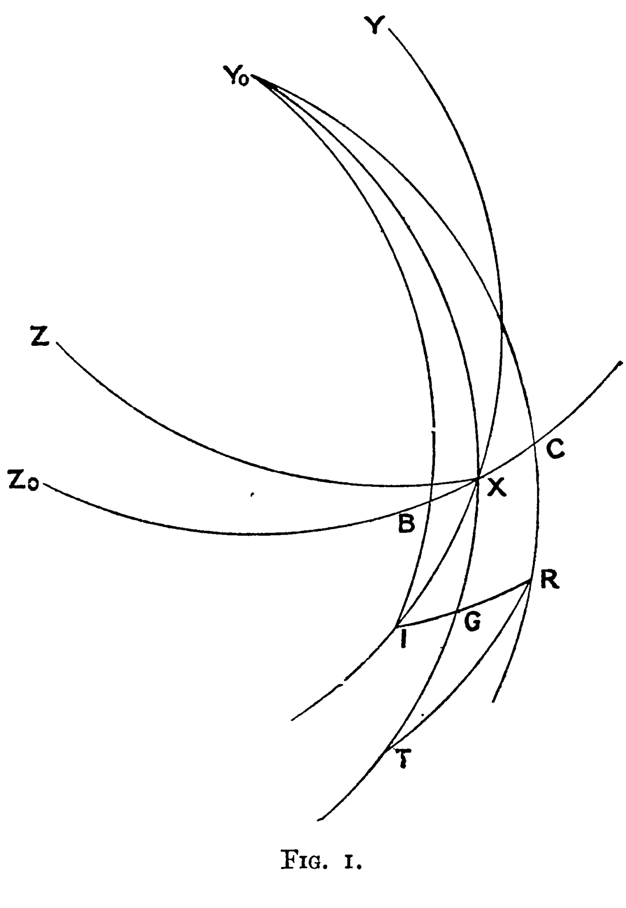

FIG. 1.

Now the point of incidence P_1 of the ray on the mirror is $(\frac{1}{4}av^2, av, 0)$, the axes being $X_0\ Y_0\ Z_0$. If then $v = 2 \tan \psi$, ψ is the angle between the axis of the mirror and the normal at P_1. We are chiefly concerned with the directions of certain lines, and we therefore construct a spherical figure (fig. 1) by drawing parallels through the centre of a sphere. Let X Y Z, $X\ Y_0\ Z_0$ correspond to the two sets of axes and I, G, R to the incident ray, the normal and the reflected ray respectively. We notice that YXI, Y_0XG, and IGR lie on the same great circles and that $IG = GR$. Let Y_0I and Y_0R cut Z_0X in B and C, and let $BX = BY_0X = \beta$ and $XC = XY_0C = \gamma$. The points B and C correspond to the projections of the incident and reflected rays on the plane XZ_0. Hence the equations of these projections are :

$$z_0 = (x - \tfrac{1}{4}av^2) \tan \beta$$
$$z_0 = (\tfrac{1}{4}av^2 - x) \tan \gamma.$$

On Y_0G produced we take T so that $GT = XG = \psi$. Then the triangles XGI, TGR are altogether equal, and consequently $RT = IX = \alpha$ and $GTR = GXI = YXY_0 = \phi$. Hence from the triangle Y_0TR, in which $Y_0T = \frac{\pi}{2} + 2\psi$ we get

$$-\sin 2\psi \cos \phi = \cos 2\psi \cot \alpha - \sin \phi \cot \gamma.$$

And from the triangle Y_0XI

$$0 = \cot \alpha - \sin \phi \cot \beta.$$

We have then

$$\tan \beta = \sin \phi \tan \alpha$$

and

$$\tan \gamma = \frac{\sin \phi}{\cos 2\psi \cot \alpha + \sin 2\psi \cos \phi}$$
$$= \frac{(4+v^2) \sin \phi \tan \alpha}{4 + 4cv - v^2}$$

We see then that General Tennant has in effect made the tacit assumption that $\gamma = \beta$. If we avoid this error and proceed with the transformation by means of the formulæ

$$y = y_0 \cos \phi - z_0 \sin \phi$$
$$z = y_0 \sin \phi + z_0 \cos \phi$$

we shall obtain the equations of the reflected ray in the proper form and arrive at results equivalent to those found in §§ 3 and 4. The oversight is most unfortunate, as it detracts from an otherwise admirable paper, which, so far as I am aware, contains the first serious discussion of the subject under consideration.

10. *On Mr. Schaeberle's Paper* (*A. J.* No. 435).—Mr. Schaeberle's paper is based on a very elementary geometrical method which does not, however, tend to simplify the results. His

notation is retained in this review of his theory. The principal formulæ are

$$\frac{x}{\mathrm{F}}=\sec^3 \tfrac{1}{2}v\left[\sin \tfrac{1}{2}v \cos \mathrm{A}+\frac{\cos v \sin \frac{1}{2}\phi \cos (180^\circ+\mathrm{A}+\mathrm{B})}{\sin (\mathrm{I}-\frac{1}{2}\phi)}\right] \quad (15\ \mathrm{S})$$

$$\frac{y}{\mathrm{F}}=\sec^3 \tfrac{1}{2}v\left[\sin \tfrac{1}{2}v \sin \mathrm{A}+\frac{\cos v \sin \frac{1}{2}\phi \sin (180^\circ+\mathrm{A}+\mathrm{B})}{\sin (\mathrm{I}-\frac{1}{2}\phi)}\right] \quad (16\ \mathrm{S})$$

to which must be added the subsidiary relations

$$\sin \mathrm{B}=\frac{\tan \theta \sin \mathrm{A}}{\sqrt{[\tan^2\frac{1}{2}v+\tan^2 \theta-2 \tan \frac{1}{2}v \tan \theta \cos \mathrm{A}]}} \quad \ldots \quad (7\ \mathrm{S})$$

$$\cos \mathrm{I}=\sin \tfrac{1}{2}v \cos \mathrm{B} \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8\ \mathrm{S})$$

$$\sin \tfrac{1}{2}\phi=\sin \mathrm{I} \cos \theta \sqrt{[\tan^2\tfrac{1}{2}v+\tan^2\theta-2 \tan \tfrac{1}{2}v \tan \theta \cos \mathrm{A}]} \quad (9\ \mathrm{S})$$

The numbers attached to these equations correspond to those given in Mr. Schaeberle's paper. His equation (7), however, contains under the radical the term $-2 \tan v \tan \theta \cos \mathrm{A}$, an apparent misprint which is corrected above. I shall now show that these equations, unpromising as they may appear, can be made to give simple expressions for the coordinates in the focal plane equivalent to those already found. It is merely necessary to eliminate B, I, and ϕ. We have, from (7 S),

$$\cos \mathrm{B}=[\tan \tfrac{1}{2}v-\tan \theta \cos \mathrm{A}]/p$$

where $$p^2=\tan^2\tfrac{1}{2}v+\tan^2\theta-2 \tan \tfrac{1}{2}v \tan \theta \cos \mathrm{A}.$$

$$\therefore \cos (\mathrm{A}+\mathrm{B})=[\tan \tfrac{1}{2}v \cos \mathrm{A}-\tan \theta]/p$$
$$\sin (\mathrm{A}+\mathrm{B})=\tan \tfrac{1}{2}v \sin \mathrm{A}/p.$$

Also, by (8S),

$$\cos \mathrm{I}=[\sin \tfrac{1}{2}v \tan \tfrac{1}{2}v-\sin \tfrac{1}{2}v \tan \theta \cos \mathrm{A}]/p$$

$$\therefore \sin \mathrm{I}=q/p.$$

where $q^2=\sin^2 \frac{1}{2}v+\tan^2\theta (1-\cos^2\mathrm{A} \sin^2 \frac{1}{2}v)-\sin v \tan \theta \cos \mathrm{A}$.

Then by (9S)

$$\sin \tfrac{1}{2}\phi=\cos \theta \,.\, q$$

$$\therefore \cos^2 \tfrac{1}{2}\phi=\cos^2\theta \cos^2 \tfrac{1}{2}v+\sin^2\theta \cos^2\mathrm{A} \sin^2\tfrac{1}{2}v+\sin v \sin \theta \cos \theta \cos \mathrm{A}$$

$$\therefore \cos \tfrac{1}{2}\phi=\cos \theta \cos \tfrac{1}{2}v+\sin \theta \sin \tfrac{1}{2}v \cos \mathrm{A}.$$

Hence

$$\sin (\mathrm{I}-\tfrac{1}{2}\phi)=[\cos \theta \cos v \sec \tfrac{1}{2}v+2 \sin \theta \sin \tfrac{1}{2}v \cos \mathrm{A}]q/p$$

$$\therefore \frac{\sin \frac{1}{2}\phi}{\sin (\mathrm{I}-\frac{1}{2}\phi)}=\frac{\cos \frac{1}{2}v \,.\, p}{\cos v(1+\tan v \tan \theta \cos \mathrm{A})}.$$

We can now substitute in (15 S). Thus:

$$\frac{x}{\mathrm{F}} = \sec^3 \tfrac{1}{2}v \left[\sin \tfrac{1}{2}v \cos \mathrm{A} - \frac{\cos \tfrac{1}{2}v (\tan \tfrac{1}{2}v \cos \mathrm{A} - \tan \theta)}{1 + \tan v \tan \theta \cos \mathrm{A}}\right]$$

$$= \frac{[\sin \tfrac{1}{2}v \tan v \tan \theta \cos^2 \mathrm{A} + \cos \tfrac{1}{2}v \tan \theta]}{1 + \tan v \tan \theta \cos \mathrm{A}} \sec^3 \tfrac{1}{2}v$$

$$\therefore x = \frac{\mathrm{F} \tan \theta}{\cos v} \cdot \frac{1 + \tan^2 \tfrac{1}{2}v \cos 2\mathrm{A}}{1 + \tan v \tan \theta \cos \mathrm{A}}.$$

And similarly, by substituting in (16 S), we get

$$\frac{y}{\mathrm{F}} = \sec^3 \tfrac{1}{2}v \left[\sin \tfrac{1}{2}v \sin \mathrm{A} - \frac{\sin \tfrac{1}{2}v \cos \mathrm{A}}{1 + \tan v \tan \theta \cos \mathrm{A}}\right]$$

$$= \sin \tfrac{1}{2}v \tan v \tan \theta \sin \mathrm{A} \cos \mathrm{A} \frac{\sec^3 \tfrac{1}{2}v}{1 + \tan v \tan \theta \cos \mathrm{A}}$$

$$\therefore y = \frac{\mathrm{F} \tan \theta}{\cos v} \cdot \frac{\tan^2 \tfrac{1}{2}v \sin 2\mathrm{A}}{1 + \tan v \tan \theta \cos \mathrm{A}}.$$

These formulæ for x and y become identical with those given for ζ and η in (2) when it is remembered that A is the angle which I have called ϕ and that θ is reckoned positive in opposite directions by Mr. Schaeberle and myself, *i.e.* that we assume the star to be on opposite sides of the axis of the mirror.

11. But though Mr. Schaeberle may claim that his formulæ are essentially correct, the same cannot be said of the figure 2 which appears on p. 19 of his paper. He has drawn two curves for $\theta = 30'$ corresponding to the zones for which $v = 5°$ and $v = 3°$. The larger curve in particular is far from accurate. With regard to the drawing of this curve the following points may be noticed: (1) it is obviously of a degree higher than the fourth, for a straight line can be drawn to cut it in six points; (2) the two loops do not cut the axis of x at right angles, but possess singular points where they cross this axis; (3) the points for which A is 90° and 270° do not lie on the axis of x; (4) the most serious error lies in the disparity between the width of the loops, the true difference being of the order 1 in 1000. With the exception of (3) the same criticisms apply also to the smaller figure. It is impossible for me to trace the cause of these errors. Possibly the supposed misprint to which I have referred is really a slip. It seems more likely that the responsibility rests with the ambiguous signs which occur in connection with the irrational expressions left in the equations. Finally it may be that the arithmetic is at fault. I give drawings of the curves (fig. 2), which I hope will be found more satisfactory. I venture to deprecate the tendency to refer to the simple geometrical aberration of the parabolic mirror as "Schaeberlian." With all proper respect for Mr. Schaeberle's work I cannot help regarding this title as unnecessary and inappropriate.

C C

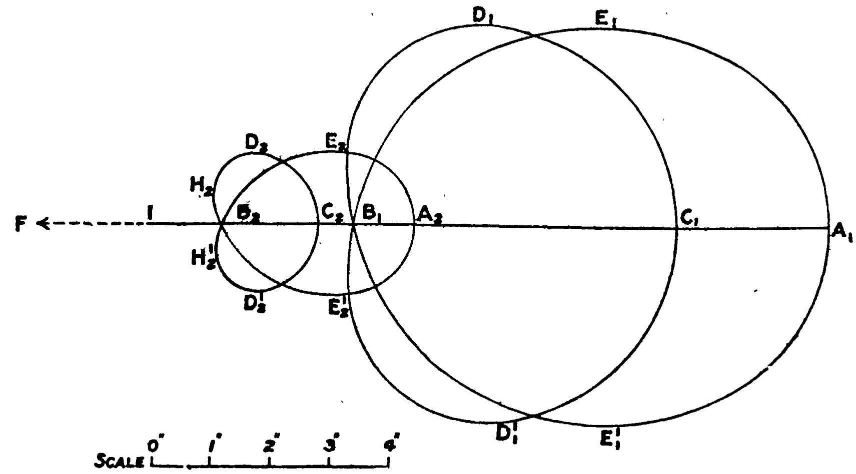

FIG. 2.

12. *On Mr. Crockett's Paper* (*Ap. J.* vii. p. 362).—I shall verify Mr. Crockett's final formulæ by deducing from them equations equivalent to (2). The set of relations marked (5) in his paper is as follows :—

$$\left.\begin{aligned} r_1 &= \frac{r \sin \alpha}{\sin (\gamma + \alpha)} \\ \cos \gamma &= \sin \theta \cos (\psi_1 - \psi) \\ \sin \gamma &= \frac{\sin (\psi_1 - \psi)}{\sin \psi} \\ \tan (\psi_1 - \psi) &= - \cos \theta \tan \psi \end{aligned}\right\} \quad \dots \quad \dots \quad (5\text{ C})$$

If I may permit myself a word of complaint I would say that the author's explanations err on the side of brevity. In the foregoing equations r, θ are the polar co-ordinates of the point of incidence in the plane containing this point and the axis of the mirror, the focus being the pole. Consequently

$$r = f \sec^2 \tfrac{1}{2}\theta.$$

Also r_1, ψ_1 are the polar co-ordinates of the corresponding point of the image in the focal plane, the focus of the mirror again being the pole. The inclination of the incident ray to the axis is α, and ψ is the same angle which I have previously denoted by ϕ. It is unnecessary to define the angle γ, since it will be eliminated immediately. Instead of the third relation we may write

$$\sin \gamma = - \cos \theta \sec \psi \cos (\psi_1 - \psi)$$

$$\therefore \sin (\gamma + \alpha) = - \sec \psi \cos (\psi_1 - \psi)$$
$$(\cos \theta \cos \alpha - \sin \theta \sin \alpha \cos \psi).$$

If then we put

$$R = -\frac{f \sec^2 \frac{1}{2}\theta \sin \alpha \cos \psi}{\cos \theta \cos \alpha - \sin \theta \sin \alpha \cos \psi}$$

we shall have

$$\begin{aligned} x_1 = r_1 \cos \psi_1 &= R \cos \psi_1 \sec (\psi_1 - \psi) \\ &= R\{\cos \psi - \sin \psi \tan (\psi_1 - \psi)\} \\ &= R \sec \psi (\cos^2 \psi + \cos \theta \sin^2 \psi) \\ &= R \sec \psi (\cos^2 \tfrac{1}{2}\theta + \sin^2 \tfrac{1}{2}\theta \cos 2\psi) \end{aligned}$$

and

$$\begin{aligned} y_1 = r_1 \sin \psi_1 &= R \sin \psi_1 \sec (\psi_1 - \psi) \\ &= R\{\sin \psi + \cos \psi \tan (\psi_1 - \psi)\} \\ &= 2R \sin \psi \sin^2 \tfrac{1}{2}\theta. \end{aligned}$$

Hence, replacing R by the expression which it denotes, we get immediately

$$x_1 = -\frac{f \tan \alpha}{\cos \theta} \cdot \frac{1 + \tan^2 \frac{1}{2}\theta \cos 2\psi}{1 - \tan \theta \tan \alpha \cos \psi}$$

$$y_1 = -\frac{f \tan \alpha}{\cos \theta} \cdot \frac{\tan^2 \frac{1}{2}\theta \sin 2\psi}{1 - \tan \theta \tan \alpha \cos \psi}$$

and these equations again, except for changes of notation, are identical with those numbered (2).

13. In figures 4 and 5 on p. 365 of his paper Mr. Crockett has given a number of curves obtained by assuming $\alpha = 1°$ and giving θ a series of values from 0° to 3°. Most of these curves have little or no significance in a practical study of the photographic instrument (see § 15), but their variations of form are theoretically interesting and I am glad that this part of the study can be completed by a reference to Mr. Crockett's work. I only wish to point out how his curves exemplify the theory of §§ 6–8. When $\theta = 3°$ (I follow here Mr. Crockett's notation) the resulting H curve is of the same type as those of fig. 2. When $\theta = 2° = 2\alpha$ the G curve possesses the triple point which results from the coincidence of the points B and C. As θ becomes smaller the point I have denoted by C, passing to the other side of B, approaches the true image for the central point of the mirror, and the F curve given by $\theta = 1\frac{1}{2}°$ has two loops on the side of C towards the centre of the field. The point C coincides with the true image when

$$\sec \theta \sec^2 \tfrac{1}{2}\theta = 1 + \tan \theta \tan \alpha$$

or approximately,

$$1 + \tfrac{1}{4}\theta^2 = 1 - \tfrac{1}{2}\theta^2 + \alpha\theta.$$

This gives $\theta = \frac{4}{3}\alpha$, and in fact when $\theta = 1\frac{1}{3}°$ the E curve is given passing through the point image given by $\theta = 0°$. Mean-

while the loops mentioned have disappeared and the corresponding value of θ can be estimated by identifying the maximum of y_1 at the point D with the minimum of x_1 at the point H. This gives $\frac{\tan\alpha}{2\tan\frac{1}{2}\theta} = \cos\frac{\pi}{4}$ or $\theta = \sqrt{2}\,.\,\alpha$ approximately. With $\alpha = 1°$, $\theta = 1° \; 24' \; 51''$, which agrees satisfactorily with Mr. Crockett's estimate of $1° \; 25'$. The complete curve for smaller values of θ consists of two loops, one of which includes the point image given by $\theta = 0°$. When $\theta = \alpha = 1°$, a case for which the D curve is drawn by Mr. Crockett, the symmetrical pair of points H, H′ where x_1 is a minimum disappear, the geometrical meaning being that at the point denoted by C the curve ceases to be concave and becomes convex to the centre of the field. Finally the point of the image nearest to the centre of the field is given by $\theta = \frac{2}{3}\alpha = 40'$, and after this point is reached the curves arising from the inner zones of the mirror contract in size and approach the point which in a certain sense is the true geometrical image. Mr. Crockett's curves are therefore in agreement with what the general theory would lead us to expect.

14. *On my previous paper* (*Astronomical Journal, No.* 435). The coordinates of a point of the image in the focal plane are given in this form:

$$\eta = -\frac{2yz\,(f+x)\sin\theta}{(4f^2-r^2)\cos\theta - 4fz\sin\theta}$$

$$\zeta = -\frac{\sin\theta\{4f^3+2fz^2-x(y^2-z^2)\}}{(4f^2-r^2)\cos\theta-4fz\sin\theta}$$

When x, y, and z are expressed in terms of f, v, and ϕ these become

$$\eta = -\frac{8\sin\phi\cos\phi\tan^2\frac{1}{2}v\sec^2\frac{1}{2}v}{4(1-\tan^2\frac{1}{2}v)-8\tan\frac{1}{2}v\tan\theta\cos\phi}\,.\,f\tan\theta$$

$$= -\frac{f\tan\theta}{\cos v}\,.\,\frac{\tan^2\frac{1}{2}v\sin 2\phi}{1-\tan v\tan\theta\cos\phi}$$

and

$$\zeta = -\frac{\{4+8\tan^2\frac{1}{2}v\cos^2\phi+4\tan^4\frac{1}{2}v\cos 2\phi\}}{4\,(1-\tan^2\frac{1}{2}v)-8\tan\frac{1}{2}v\tan\theta\cos\phi}\,.\,f\tan\theta$$

$$= -\frac{f\tan\theta}{\cos v}\,.\,\frac{1+\tan^2\frac{1}{2}v\cos 2\phi}{1-\tan v\tan\theta\cos\phi}\,.$$

These expressions are therefore, as stated, exact. In the further development of the theory the approximations employed were

$$\eta = -\frac{\tan\theta}{2f}\,yz$$

$$\zeta = -f\tan\theta - \frac{\tan\theta}{4f}\,(r^2+2z^2)$$

or, as we may now write them,

$$\eta = -f \tan\theta \,.\, \tan^2 \tfrac{1}{2}v \sin 2\phi \quad \ldots \quad \ldots \quad \ldots \quad (9)$$
$$\zeta = -f \tan\theta \{1 + \tan^2 \tfrac{1}{2}v\ (2 + \cos 2\phi)\}$$

Now the aberration contains terms of the order v^2. Hence it is in general perfectly justifiable to neglect terms of the order v^4, $v^3\theta$, or $v^2\theta^2$ for practical purposes. If this is done the approximation to η is seen to be adequate. On the other hand a proper approximation to ζ is

$$\zeta = -f \tan\theta \{1 + \tan^2 \tfrac{1}{2}v\ (2 + \cos 2\phi) + \tan v \tan\theta \cos\phi\} \quad (10)$$

which differs from the preceding expression for ζ by the addition of the term $-f \tan\theta \,.\, \tan v \tan\theta \cos\phi$.

15. The effect of this term is to make the curve of the zone image depart considerably from the two coincident circles previously found, unless θ is small compared with v. The maximum value of the term when $v = n°$ and $\theta = 1°$ is about n'', and of course it diminishes fairly rapidly towards the centre of the field. It introduces distortion apart from mere change of scale value. When $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ the term vanishes, and when ϕ is given values differing by π, the corresponding points are equidistant from the point given by (9) and the following equation. Hence the circle which is the locus corresponding to the latter equations may be regarded as a mean between the two loops represented by (9) and (10). This affords in many cases a justification for neglecting the term when v is fairly large; and when v is small, although the form of the curve as shown in the review of Mr. Crockett's paper undergoes considerable changes, the omission has little practical effect. This is due partly to the smallness of the curves and partly to the shadow which the photographic plate and its holder cast on the mirror. In the direction $\phi = \pi$ the light is evidently intercepted at least as far as the zone $v = 2\theta$. In the plane $\phi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ the zone $v = a$ is ineffective, $2a$ being the angular width of the field covered by the plate. The result is that the curves considered do not contribute much to the image, especially in the case of stars at some distance from the edge of the plate. The images are thus deprived of the fairly sharp point directed towards the centre of the field, and the geometrical focus for the centre of the mirror will in general lie nearer the focus of the surface than any point of the image actually formed. The precise effect in actual measurement can only be determined by experiment. With the reservation now stated I still consider the analysis of the distribution of intensity in the image, developed in my former paper, fairly satisfactory as a first approximation, and it is probably not worth while to push further a purely geometrical investigation. That examination seems certainly preferable to the merely general explanations

given elsewhere and will serve at least to emphasise the necessity of paying attention to the effect of star magnitude and time of exposure in the reduction of a photograph taken with a parabolic mirror.

16. *On the form of the image in a plane parallel to the focal plane.*—Let the reflected ray meet the plane $x = f - h$ in the point (ξ, η, ζ). Then

$$\eta = y + \frac{\mu}{\lambda}(f - h - x) = \eta_0 - \frac{\mu}{\lambda} . h$$

$$\zeta = z + \frac{\nu}{\lambda}(f - h - x) = \zeta_0 - \frac{\nu}{\lambda} . h,$$

where η_0, ζ_0 are the co-ordinates of the corresponding point in the focal plane already found in § 4. Now in the coefficients of h, which are of the first order in v, we may fairly neglect terms of the third order in v and θ. This will give as sufficient approximations to (1)

$$\mu/\lambda = -\sin\phi \tan v$$
$$\nu/\lambda = -\cos\phi \tan v - \tan\theta.$$

Then, keeping a consistent degree of approximation throughout, we use for η_0, ζ_0 the forms (9) and (10) of § 14. Thus

$$\eta = -f \tan\theta \tan^2 \tfrac{1}{2}v \sin 2\phi + h \tan v \sin\phi$$
$$\zeta = -f \tan\theta \{1 + \tan^2 \tfrac{1}{2}v(2 + \cos 2\phi) + \tan v \tan\theta \cos\phi\} + h(\tan\theta + \tan v \cos\phi).$$

Moreover, as quantities depending on h multiplied by terms of the third order in v and θ have already been neglected, it is allowable to put $f' = f - h$, and write

$$\left.\begin{aligned} \eta &= -f' \tan\theta \tan^2 \tfrac{1}{2}v \sin 2\phi + h \tan v \sin\phi \\ \zeta &= -f' \tan\theta \{1 + \tan^2 \tfrac{1}{2}v(2 + \cos 2\phi)\} \\ &\qquad -(f' \tan^2\theta - h) \tan v \cos\phi \end{aligned}\right\} \quad (11)$$

which become identical, as they should, with (9) and (10) when $h = 0$, and only differ from the latter by the change of scale value indicated by f' in the place of f and by the addition of the terms $h \tan v \sin\phi$ and $h \tan v \cos\phi$ to η and ζ respectively.

17. The discussion of the locus represented by (11) may fitly be combined with that of the loci represented by (9) and the following equation and by (9) and (10), for all three loci are related in an interesting way. In the first place we put

$$\rho = 2f \tan\theta \tan^2 \tfrac{1}{2}v \cos\phi \quad \ldots \quad \ldots \quad (12)$$

This is the polar equation of a circle referred to a point on its circumference, or rather of a double circle arising equally from

the variation of ϕ between 0 and π and between π and 2π. Then (9) and the following equation may be written

$$\eta = -\rho \sin \phi$$
$$\zeta = -f \tan \theta \sec^2 \tfrac{1}{2}v - \rho \cos \phi,$$

which show that the final approximations of my former paper represent a circle which, as there explained, generates a kite-shaped image of angle 60° (fig. 3*a*). Now (9) and (10) differ from the equations just considered only in the addition of the term $-f \tan^2 \theta \tan v \cos \phi$ to ζ. The nature of the modification thus introduced is easily understood, and is illustrated in fig. 3*b*. It would then be possible to deduce the character of the locus represented by (11) from the locus last considered by changing f into f' in the circular terms, $f \tan^2 \theta$ into $(f' \tan^2 \theta - h)$ in the correcting term of ζ, and finally applying to η the term $h \tan v \sin \phi$. It seems better, however, to discard the circular approximation (except as a guide to position) and to put

$$\rho = 2f' \tan \theta \tan^2 \tfrac{1}{2}v \cos \phi - h \tan v \quad \ldots \quad \ldots \quad (13)$$

This is the polar equation of a limaçon. If the term $-f' \tan^2 \theta \tan v \cos \phi$ in ζ be neglected for the moment (11) may now be written

$$\eta = -\rho \sin \phi$$
$$\zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v - \rho \cos \phi.$$

It would be tedious and unnecessary to consider possible varieties of form in detail. It is sufficient to take the case in which the limaçon is of the two-looped kind, in which case the relation to the previous double-circular approximation is obvious. The node is at the pole, which is in the same position as in the former case. The positions of two pairs of points are important. These are K, L (fig. 3*c*), given by

$$\phi = \frac{\pi}{2} \text{ and } \frac{3\pi}{2},\ \eta = \pm h \tan v,\ \zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v$$

and M, N given by

$$\phi = 0 \text{ and } \pi,\ \eta = 0,\ \zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v - 2f' \tan \theta \tan^2 \tfrac{1}{2}v \pm h \tan v$$

These points serve to fix the position of the limaçon in relation to the circle obtained by putting $h=0$.

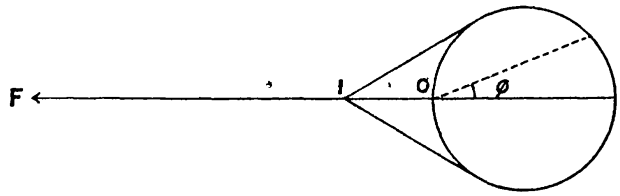

Fig. 3*a*.

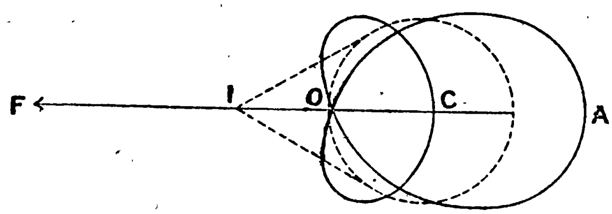

FIG. 3b.

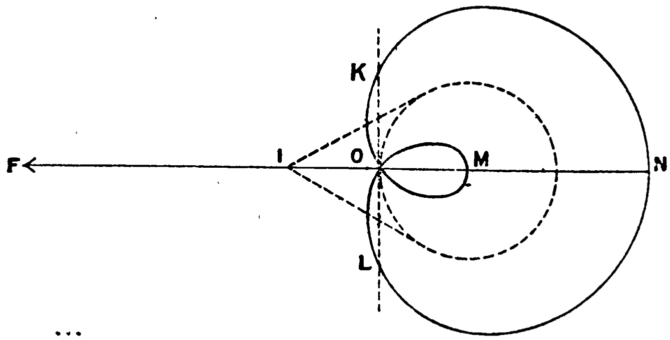

FIG. 3c.

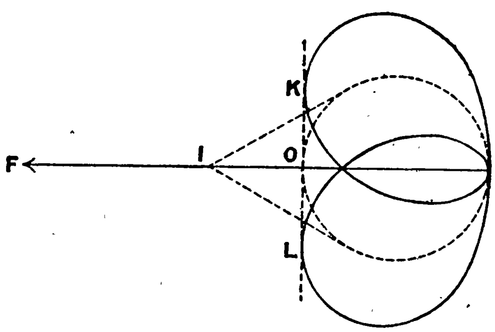

FIG. 3d.

18. In order to obtain the final approximation aimed at it is now necessary to replace the additional term in ζ, the general effect of which will be fairly evident. But it seems scarcely worth while to consider the result in detail. For it is clear that the addition will leave the points K, L where $\phi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ unaltered, and that it will cause the points M and N to move towards each other. The result is therefore very small, and in the most favourable case can only make the points M and N coincide at a point on the circumference of the circular approximation. In fact the point midway between M and N is unchanged in position. It does not seem then that a sufficient improvement in the outer parts of the field can be effected at the expense of the definition at the centre of the field and the lateral aberration of all the images, and it seems almost better to be satisfied with the best possible adjustment at the focus of the mirror. If, however,

special circumstances seem to make it desirable to make a compromise by adjusting the photographic plate so as to get the best definition, or rather the minimum radial aberration at a distance θ from the centre of the plate, then the proper position is given by

$$h = f \tan^2 \theta,$$

in which case the image is generated by a point whose coordinates are

$$\left.\begin{aligned} \eta &= -f' \tan\theta \tan^2 \tfrac{1}{2}v \sin 2\phi + f' \tan^2\theta \tan v \sin\phi \\ \zeta &= -f' \tan\theta \{1 + \tan^2 \tfrac{1}{2}v\,(2 + \cos 2\phi)\} \end{aligned}\right\} \quad \ldots \quad (14)$$

The use of curved plates seems also to offer no advantage sufficient to warrant the extra trouble and expense. The proper radius of curvature for such plates is now found to be

$$R = f^2 \tan^2 \theta / 2h = \tfrac{1}{2} f \qquad \ldots \qquad \ldots \quad (15)$$

a result which is in agreement with the well-known curvature of the field in its usual conventional sense. The impossibility of effectively overcoming the aberration of a parabolic mirror by any device of the kind considered has also been stated by Mr. S. C. Reese (*Astrophysical Journal*, vol. xii. p. 227), after an investigation of a very different kind. As the nature of the images depends on curves given by (14) a typical curve is sketched (fig. 3*d*) with the circle to which it is related in the manner already explained.

19. In conclusion a word or two of summary may be added to what has already been said in § 2 of the critical part of this paper. I still believe that my former paper (*A. J.* No. 435), which gives fig. 3*a* as a fair approximation, may be considered a satisfactory *general* explanation of the nature of the image formed by a parabolic mirror. Moreover its method and results have been of great service in the extension given to the theory in §§ 16–18 above. But confessedly accuracy has in a slight degree been sacrificed for the sake of simplicity. If then my work is to be of use in designing a parabolic mirror for a definite purpose, it will be easy to draw the necessary curves with absolute accuracy with the help of equations (2) of § 4 of the present paper. In most cases the simpler approximate forms (9) and (10) of § 14 will be sufficient. The discussion given in §§ 16–18 is necessarily brief, but it will suffice, I hope, to prove the negative result that more or less kite-shaped images are unavoidable in the case of a reflector as ordinarily adapted for photographic use.

University Observatory, Oxford:
1902 *March* 10.

The Flash Spectrum, Sumatra Eclipse, 1901 *May* 18.
By S. A. Mitchell, Ph.D.

(*Communicated by Professor J. K. Rees.*)

The transparency accompanying this paper contains:
1. Spectrum of the Second Flash.
2. Spectrum taken 5 seconds after Second Flash.
The spectra were photographed with an objective plane grating, and the positives made without enlargement.

The writer, through the courtesy of the former astronomical director of the Naval Observatory, Professor S. J. Brown, became a member of the expedition to view the Sumatra eclipse 1901 May 18.

Before reaching the island, it had been decided to occupy two stations for observations on the eclipse; one, Solok, near the central line of totality, the other Fort de Koch, near the northern edge of the Moon's shadow path.

On our arrival in the East Indies, it was soon found that the greatest trouble was going to be clouds, for almost at no time during the day was the sky perfectly clear. It was, therefore, thought best to subdivide the party at Solok, and a third station was selected at Sawah Loento, about twenty miles distant, at the terminus of the railroad, where were already Mr. and Mrs. Newall, of Cambridge, England, and a party from the Massachusetts Institute of Technology under the direction of Professor Burton.

Two instruments were taken to Sawah Loento, a camera of 104 inches focus to be used in connection with a cœlostat for photographing the corona; and a spectroscope consisting of a Rowland flat grating of 15,000 lines, with a ruled surface of $3\frac{1}{2}$ by 5 inches, and a quartz lens of $3\frac{23}{64}$ inch aperture and 72 inches focal length. Light from the Sun, reflected by the cœlostat mirror in a horizontal direction, fell on the grating, where it was diffracted and was brought to a focus on the photographic plate by means of the quartz lens. If grating and photographic plate are each perpendicular to the diffracted beam, the spectrum is "normal." It was arranged to photograph the first order spectrum from λ 3000 to λ 6000.

The final adjustments were made a few days before the eclipse by Mr. L. E. Jewell, the focusing being accomplished by means of a collimator designed by him.

The day of the eclipse dawned clear, and our hopes were that these favourable conditions would remain until after totality, which occurred shortly after noon. First contact was observed